3D Printed Models & Lagrange Mulitpliers in Calculus 3

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Multivariable Functions and Surfaces

- Calculus 3 has a large visual component.
- Students often struggle with the visualization in their head, it's not the type of visualization they are used to.
- Some topics in calc 3 are often 'glossed over' for the students. They are sometimes just given the appropriate equations and formulas and then supposed to do the calculations.
- This semester, I aimed to get my students to see more of the 'why' behind various equations and formulas

CalcPlot3D

CalcPlot3D is an excellent free resources for visualizing these surfaces and other calculus concepts:

https://c3d.libretexts.org/CalcPlot3D/index.html?

Prerequisites

Some things to remember:

- Two vectors are parallel if they are scalar multiples of each other.
- The gradient of a function $f(x, y)$ is the vector $\nabla f = \langle f_x, f_y \rangle$.
- The gradient of f is always orthogonal to it's level curve of f and it points in the direction of steepest ascent.

3D Printed Models Activity

Work in pairs(ish) with a 3D model and let's explore how to find max and mins.

Lagrange Multipliers Activity

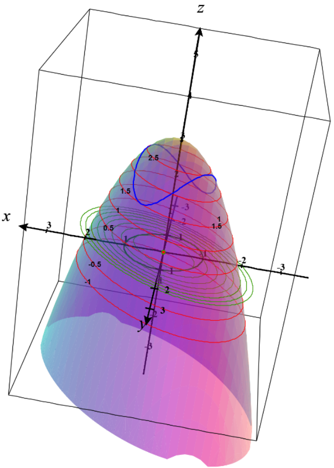
Lagrange Multipliers

Theorem

Let f be a differentiable function in a region of $\mathbb{R}^2$ that contains the smooth curve C given by $g(x, y) = 0$. Assume that f has a local extreme value (relative to values of f on C) at a point $P(a, b)$ on C. Then $\nabla f(a, b)$ is orthogonal to the line tangent to C at P and, assuming $\nabla g(a, b) \neq \vec{0}$, there is a real number λ, called **Lagrange multiplier** *such that $\nabla f(a, b) = \lambda \nabla g(a, b)$.*

Example

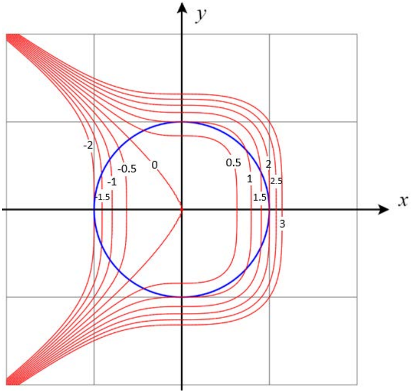
Find the max and min elevation values of the hiking trail.



CalcPlot3D Visual

Example

Consider the following Contour Map of a function $f(x, y) = 2x^3 + y^4$ and a constraint curve (hiking path) that is the unit circle. Use what we learned about Lagrange Multipliers to estimate the location on the map of the absolute max and absolute min of the function along that hiking path.



Example

CalcPlot3D Visual of previous example

Process

- Attend JMM conference where I learned how to create STL files.
- Worked with our local Architecture department and their maker space to get the 3d models printed.
- Brought them to class and had students *discover* Lagrange Multipliers!

Thank You

Thank you to the CalcPlot3D team and the course I took at JMM with: [1]

- Paul Seeburger, Monroe Community College, NY
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[1] This Lagrange Multiplier Activity was developed by Shelby Stanhope and Paul Seeburger for Taking CalcPlot3D to the Next Dimension, NSF-IUSE#2121152